QCD string in mesons and baryons

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Abstract

Field distributions generated by static $Q\bar{Q}$ and QQQ sources are calculated analytically in the framework of Gaussian (bilocal) approximation of Field Correlator Method. Special attention is paid to the QQQ system and asymmetric configurations are also studied. At large quark separations fields form distinct stringlike shape. In both cases $Q\bar{Q}$ and QQQ the string consists mainly of longitudinal color electric field. Transverse color electric component contribution is shown to be less then 3%. Baryon string has an Y-like shape with a deep well at the string junction position. Field distributions for quark-diquark and for three quarks on one line are considered. The interaction potential for quarks forming an equilateral triangle is calculated. The material of the paper is illustrated by 24 3D colored pictures.

1 Introduction

Field distributions inside the string connecting static $Q\bar{Q}$ sources have been measured repeatedly on the lattice using both connected [1-3] and disconnected [4, 5] probes. Similar measurements were done later also for Abelian projected configurations [6]. Analytic calculations for the disconnected probe made in [7] in the framework of the Gaussian approximation to the Field Correlator Method (FCM) [8, 9] have revealed a clear string-like structure of the same type as was found on the lattice. The connected probe allows to measure the sign and modulus of each field component separately.

A comparison of lattice data [1] on magnitude and direction of fields in the $Q\bar{Q}$ system with analytic predictions of FCM was done in [2], demonstrating a remarkable agreement in all distributions. In particular, the measured decrease with distance of longitudinal electric field from the string axis ("the string profile") was remarkably well described by contribution of the lowest, (bilocal) correlator [2]. It should be noted that the input of FCM is the the field correlator, defined by scalar formfactors D and D_1 [8]. The lattice measurements [10] yield for the latter the exponents with the slope $T_g \approx 0.2$fm. The dominance of the bilocal correlator (sometimes called the Gaussian Stochastic Model (GSM) of the QCD vacuum) was verified recently on the lattice by the precision measurement of static potential for $Q\bar{Q}$ Wilson loop in different SU(3) representations [11]. Analysis of data [11] made in [12] has demonstrated that GSM contribution around 99% to the static $Q\bar{Q}$ potential is consistent

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with the data. These results give an additional stimulus to the analytic calculation of field distributions using lowest bilocal correlator.

In this paper we calculate $Q\bar{Q}$ and QQQ field distributions in bilocal approximation of Field Correlator Method using connecting probes. We have found that the string consists mainly of longitudinal color electric field. For three quarks forming an equilateral triangle the string has an Y-like shape with a deep well at string junction position. We consider also field distributions for quark-diquark and for three quarks at one line cases. Besides, we calculate three-quark-interaction potential for quarks forming an equilateral triangle. The well around the string-junction position provides a decrease of slope of the interaction potential at small quark separations.

Some results of this paper were presented earlier in [13].

The paper contains 5 sections. In section 2 the $Q\bar{Q}$ longitudinal and transverse field components distributions are derived analytically and plotted in 3D pictures for different quark separations. Total field distributions including perturbative one gluon exchange are also presented. In section 3 we derive analytically three quark field distribution and plot it for different quark configurations. For three quarks forming an equilateral triangle we plot also total field including perturbative part. In section 4 baryon interaction potential is calculated. In section 5 the results are summarized and their possible physical consequences are discussed.

2 Field distributions in meson

We shall study the field distributions using gauge invariant construction $\rho_{\mu\nu}(x)$ consisting of probing plaquette $P_{\mu\nu}(x)$ and Wilson loop W connected with parallel transpoters Φ (see Fig.1). In what follows this construction is referred to as a connected probe. By definition,

$$\rho_{\mu\nu}(x) = \frac{\langle W^{\alpha}_{\beta}(x_0)\Phi^{\beta}_{\gamma}(x_0,x)(P_{\mu\nu}(x))^{\gamma}_{\delta}(\Phi^{+})^{\delta}_{\alpha}(x,x_0)\rangle}{\langle W\rangle} - 1, \tag{1}$$

where

$$W = \frac{1}{N_c}\mathrm{tr}P\exp(ig\oint_C A^a_\mu t^a dz_\mu), \tag{2}$$

$$(P_{\mu\nu}(x))^{\alpha}_{\beta} = (P\exp iga^2 F^a_{\mu\nu}(x)t^a)^{\alpha}_{\beta}, \tag{3}$$

$$\Phi^{\alpha}_{\beta}(x,y) = (P\exp ig\int_y^x A^a_\mu t^a dz_\mu)^{\alpha}_{\beta}, \tag{4}$$

$W^{\alpha}_{\beta}(x_0)$ in (1) denotes an ordered exponential along contour C of Wilson loop without the point $x_0 = (0,0,0,0)$. A rectangular contour C of size $R\times T$ lies in the plane (1 4) (since quarks are static). The surface S is bounded by the contour and has coordinates $x' = (x'_1, x'_2, x'_3, x'_4)$, where $0 \le x'_1 \le R,\ x'_2 = x'_3 \equiv 0,\ -T/2 \le x'_4 \le T/2$. The probing plaquette $P_{\mu\nu}(x)$ of size $a\times a$ is oriented in the plane $(\mu\ \nu)$. It is placed at a point $x = (x_1, x_2, x_3, x_4)$, where x_1 is the probe coordinate along the string axis, x_2 is the distance from the probe to the string axis , $x_3 = x_4 \equiv 0$. In the small-plaquette-size limit expanding (3) in powers of a^2 one obtains that the connected probe is propotional to $F(x)$:

$$\rho_{\mu\nu}(x) = iga^2\frac{\langle W^{\alpha}_{\beta}\Phi^{\beta}_{\gamma}(F^a_{\mu\nu}(x)t^a)^{\gamma}_{\delta}(\Phi^{+})^{\delta}_{\alpha}\rangle}{\langle W\rangle} + O(a^4). \tag{5}$$

Thus using the connected probe allows to measure $Q\bar{Q}$ color field components not disturbed by probing plaquette in the small a limit. We shall calculate the connected probe (1) in bilocal approximation of FCM expanding Wilson loop in $F_{\mu\nu}$ and keeping only bilocal correlators contribution. Let us write Wilson loop in terms of $F_{\mu\nu}$ using the nonabelian Stokes theorem:

$$W = \frac{1}{N_c} \exp(ig \int_S F_{\mu\nu}(x, z_0) d\sigma_{\mu\nu}(x)), \tag{6}$$

where

$$F_{\mu\nu}(x, z_0) = \Phi(z_0, x) F_{\mu\nu}(x) \Phi(x, z_0). \tag{7}$$

Averaging Wilson loop over vacuum fields in bilocal approximation one has

$$\langle W \rangle \approx \exp(-\frac{g^2}{2} \int_S \int_S d\sigma_{\mu\nu}(x) d\sigma_{\rho\sigma}(x') \frac{1}{N_c} \mathrm{tr} \langle F_{\mu\nu}(x) \Phi(x, x') F_{\rho\sigma}(x') \Phi(x', x) \rangle) \tag{8}$$

In bilocal approximation we arrived at a double surface integral. The connected probe surface S_ρ consists of Wilson loop surface S and plaquette surface S_P, $S_\rho = S + S_P$, which gives for the double integral in (8) the sum of three terms,

$$\int_{S_\rho} \int_{S_\rho} = \int_S \int_S + 2 \int_S \int_{S_P} + \int_{S_P} \int_{S_P}. \tag{9}$$

Using (9) and (8) for the connected probe (1) we realize that the first term in (9) cancels with the denominator in (1) and third term of (9) disappears in the small a limit, yielding

$$\rho_{\mu\nu}(x) \simeq \frac{\exp(-\frac{1}{2} \int_{S_\rho} \int_{S_\rho})}{\exp(-\frac{1}{2} \int_S \int_S)} - 1 =$$

$$= \exp(- \int_S \int_{S_P} - \frac{1}{2} \int_{S_P} \int_{S_P}) - 1 = - \int_S \int_{S_P} + O(a^4), \tag{10}$$

where

$$- \int_S \int_{S_P} \equiv \rho_{\mu\nu}^{\mathrm{biloc}}(x) \equiv$$

$$\equiv -a^2 \int_S d\sigma_{\rho\sigma}(x') \frac{g^2}{N_c} \mathrm{tr} \langle F_{\rho\sigma}(x') \Phi(x', x) F_{\mu\nu}(x) \Phi(x, x') \rangle. \tag{11}$$

Let us define now an averaged (colorless) field strength at point x in bilocal approximation as

$$\langle F_{\mu\nu}(x) \rangle_{Q\bar{Q}} \equiv -\frac{1}{a^2} \rho_{\mu\nu}^{\mathrm{biloc}}(x) =$$

$$= \int_S d\sigma_{\rho\sigma}(x') \frac{g^2}{N_c} \mathrm{tr} \langle F_{\rho\sigma}(x') \Phi(x', x) F_{\mu\nu}(x) \Phi(x, x') \rangle. \tag{12}$$

In FCM the following parametrization of bilocal correlators by scalar formfactors D and D_1 is suggested (see second ref. in [8]):

$$\frac{g^2}{N_c} \mathrm{tr} \langle F_{\rho\sigma}(x') \Phi(x', x) F_{\mu\nu}(x) \Phi(x, x') \rangle = (\delta_{\rho\mu} \delta_{\sigma\nu} - \delta_{\rho\nu} \delta_{\sigma\mu})(D(h^2) + D_1(h^2)) +$$

$$+ (h_\mu h_\rho \delta_{\nu\sigma} - h_\mu h_\sigma \delta_{\nu\rho} - h_\rho h_\nu \delta_{\mu\sigma} + h_\nu h_\sigma \delta_{\mu\rho}) \frac{\partial D_1(h^2)}{\partial h^2} \equiv D_{\rho\sigma,\mu\nu}(h), \tag{13}$$

where $h \equiv x - x'$.

In lattice measurements [10] both formfactors were found to be exponential beyond $x = 0.2$fm with a slope $T_g \simeq 0.2$fm:

$$D(h^2) = D(0)\exp(-\mu|h|), \quad D_1(h^2) = D_1(0)\exp(-\mu|h|), \tag{14}$$

$$D_1(0) \simeq \frac{1}{3}D(0), \quad \mu \simeq 1GeV; \quad T_g \equiv \frac{1}{\mu}.$$

We will use the form of D, D_1 (14) in the whole region of x as it was done in [2].

The values $(\rho\ \sigma)$=(1 4) on the rhs of (12) are determined by the Wilson loop orientation, and therefore the combinations of D and D_1 entering (12) are:

$$D_{14,i4}(h) = \delta_{1i}(D(h^2) + D_1(h^2)) + (h_1 h_i + h_4^2\delta_{1i})\frac{\partial D_1(h^2)}{\partial h^2}, \tag{15}$$

$$D_{14,ik}(h) = (h_k h_4 \delta_{i1} - h_i h_4 \delta_{k1})\frac{\partial D_1(h^2)}{\partial h^2}, \tag{16}$$

where $i, k = 1, 2, 3$. One can see that (15) gives color electric field components $\langle \mathbf{E}(x)\rangle_{Q\bar{Q}}$, and (16) the color magnetic ones $\langle \mathbf{B}(x)\rangle_{Q\bar{Q}}$.

Let us show that the only nonzero field components are $\langle E_1(x)\rangle_{Q\bar{Q}}$ and $\langle E_2(x)\rangle_{Q\bar{Q}}$. Really, in (15) $D_{14,34}(h) \equiv 0$ for $h_3 \equiv x_3 - x_3' \equiv 0$, so that in (12) $\langle E_3(x)\rangle_{Q\bar{Q}} \equiv \langle F_{34}(x)\rangle_{Q\bar{Q}} \equiv 0$. $D_{14,ik}(h)$ in (16) is antisymmetric in $h_4 \equiv -x_4'$, so after integration in (12) over $\int_S d\sigma_{\rho\sigma}(x') = \int_0^R dx_1' \int_{-T/2}^{T/2} dx_4'$ one obtains $\langle \mathbf{B}(x)\rangle_{Q\bar{Q}} \equiv 0$.

Let us calculate $\langle E_1(x)\rangle_{Q\bar{Q}}$ and $\langle E_2(x)\rangle_{Q\bar{Q}}$.

$$\langle E_1(x)\rangle_{Q\bar{Q}} = \int_0^R dx_1' \int_{-T/2}^{T/2} dx_4' \left(D(h^2) + D_1(h^2) + (h_1^2 + h_4^2)\frac{\partial D_1(h^2)}{\partial h^2}\right) \equiv \tag{17}$$

$$\langle E_1(x)\rangle_{Q\bar{Q}}^{D} + \langle E_1(x)\rangle_{Q\bar{Q}}^{D_1}.$$

At $T \to \infty$

$$\langle E_1(x)\rangle_{Q\bar{Q}}^{D} = D(0)\int_{x_1-R}^{x_1} dh_1 \int_{-\infty}^{\infty} dh_4 \exp(-\mu\sqrt{h_1^2 + h_2^2 + h_4^2}) =$$

$$2D(0)\int_{x_1-R}^{x_1} dx \sqrt{x^2 + x_2^2} K_1(\mu\sqrt{x^2 + x_2^2}), \tag{18}$$

where K_1 is McDonald function.

$$\langle E_1(x)\rangle_{Q\bar{Q}}^{D_1} = \int_{x_1-R}^{x_1} dh_1 \int_{-\infty}^{\infty} dh_4 \frac{1}{2}\left(\frac{\partial h_1 D_1}{\partial h_1} + \frac{\partial h_4 D_1}{\partial h_4}\right) =$$

$$\frac{D(0)}{3}\int_0^{\infty} dh_4 \left(x_1 \exp(-\mu\sqrt{x_1^2 + x_2^2 + h_4^2}) - (x_1 - R)\exp(-\mu\sqrt{(x_1-R)^2 + x_2^2 + h_4^2})\right) =$$

$$\frac{D(0)}{3}\left(x_1\sqrt{x_1^2 + x_2^2} K_1(\mu\sqrt{x_1^2 + x_2^2}) - (x_1 - R)\sqrt{(x_1-R)^2 + x_2^2} K_1(\mu\sqrt{(x_1-R)^2 + x_2^2})\right). \tag{19}$$

$$\langle E_2(x)\rangle_{Q\bar{Q}} = \int_0^R dx_1' \int_{-T/2}^{T/2} dx_4' h_1 h_2 \frac{\partial D_1(h^2)}{\partial h^2} = \int_{x_1-R}^{x_1} dh_1 \int_{-\infty}^{\infty} dh_4 \frac{h_2}{2}\frac{\partial D_1}{\partial h_1} =$$

$$\frac{D(0)x_2}{3}\int_0^\infty dh_4\left(\exp(-\mu\sqrt{x_1^2+x_2^2+h_4^2})-\exp(-\mu\sqrt{(x_1-R)^2+x_2^2+h_4^2})\right)=$$
$$\frac{D(0)x_2}{3}\left(\sqrt{x_1^2+x_2^2}K_1(\mu\sqrt{x_1^2+x_2^2})-\sqrt{(x_1-R)^2+x_2^2}K_1(\mu\sqrt{(x_1-R)^2+x_2^2})\right) \tag{20}$$

Let us proceed with some evaluations. At $R\gg T_g$

$$E_1^D(R/2,0)=4D(0)\int_0^{R/2}dxxK_1(x/T_g)=2\pi D(0)T_g^2 \tag{21}$$

At quark position

$$E_1^D(0,0)=2D(0)\int_0^{R}dxxK_1(x/T_g)=\pi D(0)T_g^2\equiv\sigma, \tag{22}$$

where σ=0.9GeV/fm is the $Q\bar{Q}$ string tension. The same value of σ one has in the bilocal approximation of FCM from area law of Wilson loop.

The fields E_2 and $E_1^{D_1}$ are maximal at $x\approx T_g$,

$$E_2(0,T_g)=E_1^{D_1}(T_g,0)=\frac{D(0)}{3}T_g^2K_1(1)=\frac{0.38\pi}{6}D(0)T_g^2 \tag{23}$$

We conclude that $E_2^{\max}/E_1^{\max}\approx 0.38/12=3.2\%$.

Perturbative interaction in the leading α_s order is defined by the one gluon exchange between quarks which leads to the Coulomb-type field contribution,

$$\mathbf{E}^{\rm Coul}=C_F\alpha_s\left(\frac{\mathbf{r}_1}{r_1^3}-\frac{\mathbf{r}_2}{r_2^3}\right), \tag{24}$$

where $\mathbf{r}_1$ is a vector from the quark to the probe, and $\mathbf{r}_2$ is from the antiquark to the probe. For fundamental quarks and $Q\bar{Q}$ in $SU(3)$ singlet the Casimir operator $C_F={\rm tr}t^at^a=\frac{4}{3}$. The parameter $e=\frac{4}{3}\alpha_s=0.295$ is defined using the lattice mesurements of $Q\bar{Q}$ potential fitted by Cornell potential $V_{\rm Corn}=-\frac{e}{R}+\sigma R$ (see review [14] and references therein).

In Figs.2–5(a)[1] we plot $\langle E_1(x_1,x_2)\rangle^2_{Q\bar{Q}}$ distributions for quark separations $R=T_g,5T_g,10T_g$ $30T_g$. One can observe in the pictures how the string of a characteristic shape between quark and antiquark is forming starting from $R=5T_g$. On Figs. 2–5(b) the total field distributions with perturbative one gluon exchange included are plotted for the same separations,

$$(\mathbf{E}^{\rm tot}_{Q\bar{Q}})^2\equiv(E_1^{\rm Coul}+\langle E_1\rangle_{Q\bar{Q}})^2+(E_2^{\rm Coul})^2. \tag{25}$$

One can see that the one gluon exchange dominates at separations smaller then $T_g=0.2$fm.

In Fig.6 $\langle E_2(x_1,x_2)\rangle^2_{Q\bar{Q}}$ distributions for $R=T_g$ and $30T_g$ are presented. One can prove that E_2 does not create by itself the string. The magnitude of E_2 at distances of the order of T_g around the quark and antiquark positions is less then 3% of $E_1^{\max}$ and is decreasing fast with distance from Q ($\bar{Q}$).

In Fig.7 are presented the transverse crossections of distributions at fixed $x_1=R/2$ plotted in Figs.2–5(a). For $R=10T_g$ and $30T_g$ the string profiles practically coincide and at $R\geq 10T_g$ the string acquires its saturation form. The field magnitude in the middle of string is $E^{\rm sat}\equiv E_1^{\max}=1.8GeV/fm=2\sigma$. The string width is $\delta x^{\rm sat}=2.2T_g$ (by definition of $\delta x^{\rm sat}$, $E_1^2(R/2,\delta x^{\rm sat}/2)=1/2(E^{\rm sat})^2$).

[1] You can get quality ps figures from http://heron.itep.ru/∼kuzmenko/figures.tar.gz

In Fig.8 a longitudinal string crossection at fixed $x_2 = 0$ for $R = 30T_g$ is shown. The field is increasing rapidly in the range from $-3T_g$ to $3T_g$ and then is forming the long plateau $E_1 = E^{\rm sat}$. In the same figure the total field with perturbative one gluon exchange included is shown. It is decreasing monotonically at $x_1 > 0$, flattens at $x_1 \simeq 5T_g$ and is increasing at $25T_g < x_1 < 30T_g$; in the whole range of $0 < x_1 < R$ sign of derivative does not change.

In Fig.9 the field distributions $\langle E_1(x_1, x_2)\rangle^{D_1}_{Q\bar{Q}}$ are presented for $R = T_g$ and $30T_g$. Just as E_2, $E_1^{D_1}$ does also not create by itself the string. Magnitude of $E_1^{D_1}$ at distances $\sim T_g$ around quark and antiquark positions is less then 3% $E^{\rm sat}$ and is decreasing fast off the quark (antiquark). One can distinguish two symmetrical distributions around quark and antiquark. At $R = T_g$ they are superposed and the distribution has a maximum at zero. At $R = 30T_g$ a region in which the field is absent appears between them. These distributions are antisymmetrical under transformations $x_1 \to -x_1$ and $(x_1 - R) \to -(x_1 - R)$ respectively.

3 Field distributions in baryons

In this section we study field distributions generated by three static quarks in different configurations. Let us first consider three quarks forming an equilateral triangle. The contour of baryon Wilson loop $W^{(3Q)}$ consists of three contours C_Γ, $\Gamma = A, B, C$, formed of quark trajectories, and the string junction trajectory (see Fig.10). The string junction position is determined by the minimal area condition. In Fig.10 three contour lopes are intersecting at angles $\frac{2\pi}{3}$, forming a Mercedes star. The gauge invariance of the loop is provided by the antisymmetric tensors:

$$W^{(3Q)} = \frac{1}{3}\epsilon_{\alpha\beta\gamma}\Phi^\alpha_{\alpha'}(C_A)\Phi^\beta_{\beta'}(C_B)\Phi^\gamma_{\gamma'}(C_C)\epsilon^{\alpha'\beta'\gamma'} \tag{26}$$

The probing plaquette $P_{\mu\nu}(x)$ is attached by parallel transporters to the contour C_A at a point $x^0 : x^0_1 = x^0_4 = 0,\ x^0_3 = R$. For the entire connected probe construction the coordinate $x_2 \equiv 0$.

In the expression of averaged field in the bilocal FCM approximation (12) we are now to sum over three surfaces A, B, C :

$$\langle F_{\mu\nu}(x)\rangle_{3Q} = \sum_{\Gamma=A,B,C}\int_{\Sigma_\Gamma} d\sigma^\Gamma_{\rho\sigma}(x')\frac{g^2}{N_c}{\rm tr}\langle F_{\rho\sigma}(x')\Phi(x',x)F_{\mu\nu}(x)\Phi(x,x')\rangle. \tag{27}$$

Here Σ_Γ is the surface corresponding to the contour C_Γ, $\Gamma = A, B, C$; the sides of the rectangular surface have an extention R and T. Introducing in the plane (x_1, x_3) unit vectors $\mathbf{n}^A = (0,1), \mathbf{n}^B = (\frac{\sqrt{3}}{2}, -\frac{1}{2}), \mathbf{n}^C = (\frac{-\sqrt{3}}{2}, -\frac{1}{2})$, we integrate in (27) over the surfaces:

$$d\sigma^\Gamma_{\rho\sigma}(x')F_{\rho\sigma}(x') = d\sigma^\Gamma_{i4}(x')F_{i4}(x') = n^\Gamma_i E_i(l'\mathbf{n}^\Gamma, x'_4)dl'dx'_4 \tag{28}$$

with the result

$$\langle F_{\mu\nu}(x)\rangle_{3Q} = \sum_\Gamma n^\Gamma_i \int_0^R dl' \int_{-T/2}^{T/2} dx'_4 D_{i4,\mu\nu}(l'\mathbf{n}^\Gamma - \mathbf{x}, x'_4 - x_4). \tag{29}$$

In what follows we shall only consider the D contribution which, as was demonstrated in previous section , gives the string form. From (29),(15),(14) one obtains

$$\langle E_3(x_1, x_3)\rangle_{3Q} = D(0)\int_0^R dl' \int_{-T/2}^{T/2} dx'_4 (e^{-\mu\sqrt{x_1^2+(x_3-l')^2+x_4'^2}} -$$

$$-\frac{1}{2}e^{-\mu\sqrt{(x_1-\frac{\sqrt{3}}{2}l')^2+(x_3+l'/2)^2+x_4'^2}}-\frac{1}{2}e^{-\mu\sqrt{(x_1+\frac{\sqrt{3}}{2}l')^2+(x_3+l'/2)^2+x_4'^2}}), \tag{30}$$

$$\langle E_1(x_1,x_3)\rangle_{3Q}=D(0)\int_0^R dl'\int_{-T/2}^{T/2}dx_4'(\frac{\sqrt{3}}{2}e^{-\mu\sqrt{(x_1-\frac{\sqrt{3}}{2}l')^2+(x_3+l'/2)^2+x_4'^2}}-$$

$$-\frac{\sqrt{3}}{2}e^{-\mu\sqrt{(x_1+\frac{\sqrt{3}}{2}l')^2+(x_3+l'/2)^2+x_4'^2}}). \tag{31}$$

As one can see from (30),(31), the baryon string is a superposition of the meson string $\langle E_1(x_3,x_1)\rangle^D_{Q\bar{Q}}$ given in (18) and two such meson strings obtained by rotation over angles $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$:

$$\langle E_3(x_1,x_3)\rangle_{3Q}=\langle E_1(x_3,x_1)\rangle^D_{Q\bar{Q}}-\frac{1}{2}\langle E_1(x_3',x_1')\rangle^D_{Q\bar{Q}}-\frac{1}{2}\langle E_1(x_3'',x_1'')\rangle^D_{Q\bar{Q}}, \tag{32}$$

$$\langle E_1(x_1,x_3)\rangle_{3Q}=\frac{\sqrt{3}}{2}\langle E_1(x_3',x_1')\rangle^D_{Q\bar{Q}}-\frac{\sqrt{3}}{2}\langle E_1(x_3'',x_1'')\rangle^D_{Q\bar{Q}}, \tag{33}$$

where

$$x_1'=-\frac{1}{2}x_1-\frac{\sqrt{3}}{2}x_3,\quad x_3'=\frac{\sqrt{3}}{2}x_1-\frac{1}{2}x_3; \tag{34}$$

$$x_1''=-\frac{1}{2}x_1+\frac{\sqrt{3}}{2}x_3,\quad x_3''=-\frac{\sqrt{3}}{2}x_1-\frac{1}{2}x_3.$$

In Figs.11–13(a) and Fig.14 the squared field distribution is plotted

$$\langle \mathbf{E}(x_1,x_3)\rangle^2_{3Q}=\langle E_1(x_1,x_3)\rangle^2_{3Q}+\langle E_2(x_1,x_3)\rangle^2_{3Q} \tag{35}$$

for $R=T_g,5T_g,10T_g$ and $30T_g$. A characteristic feature of all distributions is a deep well around the string junction position. At the junction position itself the field vanishes, since for the symmetry reason at this point a preferred direction is absent. Outside the well the baryon string becomes a sum of three meson strings going from the quarks to the string junction. Let us note that field distribution in the region around string junction will remain intact even for different distances from quarks to string junction were all the "outer" meson strings saturated.

The Coulomb field of one gluon exchange for three quarks is

$$\mathbf{E}^{\rm Coul}_{3Q}=-\frac{C_F\alpha_s}{2}\left(\frac{\mathbf{r}_1}{r_1^3}+\frac{\mathbf{r}_2}{r_2^3}+\frac{\mathbf{r}_3}{r_3^3}\right), \tag{36}$$

where $\mathbf{r}_i$ is distance from the i-th quark to the probe. Factor $\frac{C_F}{2}$ appears due to contraction of antisymmetric tensors in Wilson loop (26):

$$\frac{1}{3!}\epsilon_{\alpha\beta\gamma}\epsilon_{\alpha\sigma\tau}t^a_{\beta\sigma}t^a_{\gamma\tau}=-\frac{1}{6}{\rm tr}(t^at^a)=-\frac{C_F}{2}, \tag{37}$$

where $C_F=\frac{4}{3}$ is Casimir operator; $C_F\alpha_s=0.295$ (see explanation after (24)). The total field

$$\mathbf{E}^{\rm tot}_{3Q}=\langle\mathbf{E}\rangle_{3Q}+\mathbf{E}^{\rm Coul}_{3Q} \tag{38}$$

is given in Figs.11–13(b) and Fig.15. In Fig.11(b) the total field is at least two orders of magnitude greater than the nonperturbative part (Fig.11(a)) in entire region considered. In

Fig.12(b) we still see a half of string, going out of a Coulomb spike. In Fig.13(b) and Fig.(15) the Coulomb field only changes the shape of the adjacent piece of the string and does not disturb the rest of it.

In Fig.16 are shown the longitudinal crossections of strings along x_3 axis at $x_1 = 0$ depicted in Figs.11–13(a), 14. At $R \geq 5T_g$ the shape of the well does not depend on quark separation. At $R \geq 10T_g$ the string becomes saturated; its longitudinal crossection at $0 \leq x_3 \leq 6T_g$ rises from 0 to $(E^{\rm sat})^2$ and then levels up. In the negative x_3 region $0 \leq |x_3| \leq 1.25T_g$ the crossection grows from 0 to $0.25{\rm GeV}^2/{\rm fm}^2$ and decreases rapidly at $1.25T_g \leq |x_3| \leq 5T_g$. Let us define a well radius $R_{\rm well}$ as $E^2(0, R_{\rm well}) = (E^{\rm sat})^2/2$. Then from the Fig.16 we find $R_{\rm well} = 1.75T_g$.

Let us consider now a quark-diquark configuration when two quarks are close to each other and far from the third one. We take them to form an isosceles triangle with a base much shorter than its lateral sides. The string is formed along the minimal path, consistsing of three segments connecting quarks with string junction and intersecting at angles $\frac{2\pi}{3}$, like a Mercedes star. Let us denote R_{QQ} the length of two short segments and R_Q that of the long one. In Eq.(29) to calculate the field distributions one can now integrate over dl' in the range from 0 to R_Q for $\Gamma = A$ and from 0 to R_{QQ} for $\Gamma = B, C$. Adding to Eqs.(32),(33) the indices corresponding to integration ranges, one has

$$\langle E_3(x_1,x_3)\rangle_{Q-QQ} = \langle E_1(x_3,x_1)\rangle^{D,R_Q}_{Q\bar{Q}} - \frac{1}{2}\langle E_1(x'_3,x'_1)\rangle^{D,R_{QQ}}_{Q\bar{Q}} - - \frac{1}{2}\langle E_1(x''_3,x''_1)\rangle^{D,R_{QQ}}_{Q\bar{Q}}, \tag{39}$$

$$\langle E_1(x_1,x_3)\rangle_{Q-QQ} = \frac{\sqrt{3}}{2}\langle E_1(x'_3,x'_1)\rangle^{D,R_{QQ}}_{Q\bar{Q}} - \frac{\sqrt{3}}{2}\langle E_1(x''_3,x''_1)\rangle^{D,R_{QQ}}_{Q\bar{Q}}. \tag{40}$$

In Fig.17 we plot distributions

$$\langle \mathbf{E}(x_1,x_3)\rangle^2_{Q-QQ} = \langle E_1(x_1,x_3)\rangle^2_{Q-QQ} + \langle E_3(x_1,x_3)\rangle^2_{Q-QQ} \tag{41}$$

for $R_Q = 30T_g$ and two values of R_{QQ}, (a): $R_{QQ} = 0.5T_g$, (b): $R_{QQ} = 3T_g$. In case (a) one observes a mesonic string (cf. Fig.5(a)). In case (b) one can see a baryonic string with the well in the string junction area. How this transition happens? In accordance with (39),(40) the field strength at $x_1 = x_3 = 0$ is determined by the difference of the field values of mesonic strings of length R_Q R_{QQ} at this point. The field strength of the mesonic string (18) $E_0(R)$ at the origin for the string of length R is

$$E_0(R) = \frac{2\sigma}{\pi}\int_0^{R/T_g} K_1(x)x dx. \tag{42}$$

This function increases linearly from zero at small R and saturates to the asymptotic value $E_{\rm asymp} = \sigma$ at $R > 4T_g$. The dependence $E_0^2(R)$ is shown in Fig.18.

Let us define a radius of transition of baryonic string into mesonic one $R_{\rm bar\to mes}$ as $E_0^2(R_{\rm bar\to mes}) = E^2_{\rm asymp}/2$. From Fig.18 we find $R_{\rm bar\to mes} = 1.5T_g$. At $R_{QQ} > R_{\rm bar\to mes}$ the quark-diquark string has a characteristic baryonic form with the well at the string junction (the case of Fig.17(b)) and at $R_{QQ} < R_{\rm bar\to mes}$ it turns into a mesonic one (the case of Fig.17(a)).

Let us now consider the case when quarks are placed along one line at distances R_1 and R_2 between them. The string junction position, as in previous cases, is defined by the minimal

string length condition. For given quark positions the string length is minimal when the string junction position coincides with that of a middle quark (placed between two others) and is equal to the sum of distances from this quark to others.

The contour of Wilson loop is in the plane (1 4). Proceeding just as in the beginning of section, we introduce in the plane (1 3) two unit vectors $\mathbf{n}_1 = (-1,0), \mathbf{n}_2 = (1,0)$ and get

$$\langle E_3(x_1,x_3)\rangle_{3Ql} \equiv 0, \tag{43}$$

$$\begin{aligned}\langle E_1(x_1,x_3)\rangle_{3Ql} = -\int_0^{R_1} dl' \int_{-T/2}^{T/2} dx_4' D_{14,14}(-l'-x_1,-x_3,x_4')+\\ +\int_0^{R_2} dl' \int_{-T/2}^{T/2} dx_4' D_{14,14}(l'-x_1,-x_3,x_4') =\\ = -\langle E_1(-x_1,x_3)\rangle_{Q\bar{Q}}^{D,R_1} + \langle E_1(x_1,x_3)\rangle_{Q\bar{Q}}^{D,R_2}.\end{aligned} \tag{44}$$

Distributions $\langle E_1(x_1,x_3)\rangle_{3Ql}^2$ for $R_1 = R_2 = 15T_g$ and $R_1 = 10T_g, R_2 = 20T_g$ are shown in Figs.19 (a),(b). Since both mesonic strings have saturated profiles, the field at the string junction position $x_1 = 0$ is exactly zero.

4 Quark interaction potential in baryon

In this section we consider static quarks placed in vertices of an equilateral triangle at distance R from the string junction.

As the Wilson loop is a Green function of three static quarks, it gives the quark interaction potential in the baryon,

$$V^{(3Q)}(R) = -\lim_{T\to\infty}\frac{1}{T}\ln\langle W^{(3Q)}(R,T)\rangle. \tag{45}$$

Using bilocal approximation we get an expression of potential as a sum of surface integrals of bilocal correlators over Wilson loop surface. We integrate over the surfaces just as in calculation of averaged baryon field.

$$\begin{aligned}-\langle W^{(3Q)}(R,T)\rangle = \frac{1}{2}\sum_{a,b=A,B,C}\int_{\Sigma_a}\int_{\Sigma_b} d\sigma^a_{\mu\nu}(x)d\sigma^b_{\rho\sigma}(x')\times\\ \times\frac{g^2}{N_c}\mathrm{tr}\langle F_{\mu\nu}(x)\Phi(x,x')F_{\rho\sigma}(x')\Phi(x',x)F_{\mu\nu}(x)\rangle =\\ = \frac{3}{2}\sum_{b=A,B,C} n_i^A n_k^b \int_0^R\int_0^R dl dl' \int_{-T/2}^{T/2}\int_{-T/2}^{T/2} dx_4 dx_4' D_{i4,k4}(l\mathbf{n}^A - l'\mathbf{n}^b, x_4 - x_4').\end{aligned} \tag{46}$$

In the last equation of (46) we have used symmetry of the Wilson loop contour –

$$\sum_{a,b=A,B,C} = 3\sum_{a=A;\ b=A,B,C}. \tag{47}$$

As in the previous section we shall only consider the contribution to the potential from the formfactor D. Then

$$V^{(3Q)}(R) = \lim_{T\to\infty}\frac{3}{2}\frac{D(0)}{T}\int_0^R\int_0^R dl dl' \int_{-T/2}^{T/2}\int_{-T/2}^{T/2} dx_4 dx_4' \{e^{-\mu\sqrt{(l-l')^2+(x_4-x_4')^2}} -$$

$$- e^{-\mu\sqrt{\frac{3}{4}l'^2+(l+l'/2)^2+(x_4-x_4')^2}}\}. \tag{48}$$

Let us call the first exponential in (48) a "diagonal" as it is obtained at $a = b = A$ in (46) and second one — "nondiagonal" as it is obtained at $a = A,\ b = B, C$ in (46). Integrating in (48) one obtains

$$V^{(3Q)}(R) = \frac{6\sigma\mu^2}{\pi}\{R\int_0^R dl l K_1(\mu l) - \frac{1}{\mu^3}(2-(\mu R)^2K_2(\mu R))-$$

$$-\frac{1}{2}\int_0^R dl \int_0^R dl'\sqrt{\frac{3}{4}l'^2+(l+l'/2)^2}K_1(\mu\sqrt{\frac{3}{4}l'^2+(l+l'/2)^2})\}, \tag{49}$$

where we have used the normalization of $D(0)$ (22).

First two terms of (49) result from the diagonal and the third one from the nondiagonal exponentials of (48). The first term yields linear potential with a slope 3σ at distances $R \gg T_g$, the second is a small correction to the first one (at large R it equals $12\sigma T_g/\pi$). The third term is increasing and significantly contributing to the potential growth till $R \simeq 3T_g$ and then flattens. This term is due to the well in the middle of baryonic string.

Potential behaviour $V^{(3Q)}(R)$ is shown in Fig.20(a). Tangent slope $\sigma'(R) \equiv \frac{dV}{dR}(R)$ increases from zero to 3σ in the range of $0 \le R \le 6T_g$. Beyond this range one can consider baryonic string as a sum of three mesonic ones with the radius of well is subtracted (cf. Fig.8 and Fig.16).

It Fig.20(b) the total potential $V^{(3Q)}_{\rm tot}(R)$ is presented with the one gluon exchange included,

$$V^{(3Q)}_{\rm tot}(R) = -\frac{C_F\alpha_s}{2}\sum\frac{1}{r_{ij}} + V^{(3Q)}(R) = -\frac{\sqrt{3}}{2}\frac{C_F\alpha_s}{R} + V^{(3Q)}(R). \tag{50}$$

$r_{ij} = R\sqrt{3}$ is quark separation, summation is performed over three pairs, $C_F\alpha_s = 0.295$ (see explanations after (24), (36)). In the figure one can see a change of the potential slope as a consequence of the well in the middle of baryonic string.

Let us compare results of our calculations with lattice measurements of baryonic potential (review [14] and refs. therein). In the latter the potential for quarks forming an equilateral triangle was fitted by a Coulomb plus linear (Cornell) potential in the range of $0.055\text{fm} \le R \le 0.71\text{fm}$. As a result the slope of the linear potential was found to be 2.6σ which is equal to the $\sigma'(3.5T_g)$. Thus the effect of the potential decrease due to the well in the middle of baryon string is consistent with the effect obtained on the lattice.

5 Summary

We have calculated connected-probe field distributions for $Q\bar{Q}$ and QQQ cases using the lowest (Gaussian) field correlator. Since the Gaussian correlator contributes 99% to the static $Q\bar{Q}$ potential [11,12], one expects that higher correlators would not change our picture significantly.

The connected-probe analysis allows to distinguish directions of components of color electric and color magnetic field due to probe orientation. One can see that the string mainly consists of the longitudinal color electric field. The transverse electric component contributes less than 3% of the longitudinal one. The magnitude of longitudinal component is given by the formfactor D and the transverse one by D_1. The D_1 contribution to longitudinal

component is less than 3%. At separations R greater then $10T_g$ the $Q\bar{Q}$ string profile becomes saturated and does not change with increase of R. The saturated string width is equal to $2.2T_g$. Our $Q\bar{Q}$ results are in agreement with earlier calculations in [1-3]. The bulk of the string fields is also in accordance with disconnected-probe analyses [4, 5].

The baryon string picture is obtained in our paper for the first time. A deep well in the electric field distribution appears around the string-junction position, where electric field vanishes because of symmetry arguments. The well has a radius of $1.75T_g$ demonstrating strongly suppressed fields in the middle of the heavy baryon. For quark-diquark configuration we have defined the radius of baryonic to mesonic string transition, which was found to be equal to $1.5T_g$. For three quarks on one line the field at the middle quark position was found to vanish when quark separations are large. Since the Wilson loop for the QQQ configuration used to generate interaction is also applicable for light quarks [15], physical consequences of this well can be in principle observable both for light and heavy hadrons. One consequence can be illustrated by Fig.20, where the nonperturbative part of the potential grows very slowly at small R, so that the asymptotic slope is obtained only at very large distances. Therefore an effective slope for ground-state hadrons can be some 10-20% smaller, the fact which is in agreement with relativistic quark model of baryons [16] and with recent lattice calculations of static QQQ potential [14].

One should note that the vanishing of fields at the string junction holds for the connected-probe analysis of each separate field component, and may not be true for field fluctuations at this point, measured by the disconnected probe.

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Figure 1: A connected probe for $Q\bar{Q}$.

Figure 2: $\langle E_1(x_1,x_2)\rangle^2_{Q\bar{Q}}$ (a) and $(\mathbf{E}^{\rm tot}_{Q\bar{Q}}(x_1,x_2))^2$ (b) distributions in GeV^2/fm^2. Quark separation $R = T_g$. Both x_1 and x_2 are measured in T_g. Q and $\bar{Q}$ positions are marked with points.

Figure 3: $\langle E_1(x_1,x_2)\rangle^2_{Q\bar{Q}}$ (a) and $(\mathbf{E}^{\rm tot}_{Q\bar{Q}}(x_1,x_2))^2$ (b) distributions in GeV^2/fm^2. $R = 5T_g$. Both x_1 and x_2 are measured in T_g. Q and $\bar{Q}$ positions are marked with points and vertical lines.

Figure 4: The same as in Fig.3 but for $R = 10T_g$.

Figure 5: The same as in Fig.3 but for $R = 30T_g$.

Figure 6: $\langle E_2(x_1,x_2)\rangle_{Q\bar{Q}}$ distribution in GeV/fm for $R = T_g$ (a) and $R = 30T_g$ (b). Both x_1 and x_2 are measured in T_g. Q and $\bar{Q}$ positions are marked with points and vertical lines.

Figure 7: $Q\bar{Q}$ string transverse crossection profile $\langle E_1(R/2,x_2)\rangle^2_{Q\bar{Q}}$ in GeV^2/fm^2. x_2 is measured in T_g. Dotted, dashed, long dashed and solid lines refer to $R = T_g, 5T_g, 10T_g$ and $30T_g$. At R larger then $10T_g$ the string becomes saturated; $R = 10T_g$ and $R = 30T_g$ profiles practically coincide.

Figure 8: $\langle E_1(x_1,0)\rangle^2_{Q\bar{Q}}$ (solid line) and $(\mathbf{E}^{\rm tot}_{Q\bar{Q}}(x_1,0))^2$ (dashed line) in GeV^2/fm^2. x_1 is measured in T_g. $R = 30T_g$.

Figure 9: D_1 contribution to $\langle E_1(x_1,x_2)\rangle_{Q\bar{Q}}$ distribution in GeV/fm for $R = T_g$ (a) and $R = 30T_g$ (b). Both x_1 and x_2 are measured in T_g. Q and $\bar{Q}$ positions are marked with points and vertical lines.

Figure 10: A connected probe for QQQ.

Figure 11: $\langle \mathbf{E}(x_1,x_3)\rangle^2_{3Q}$ (a) and $(\mathbf{E}^{\rm tot}_{3Q}(x_1,x_3))^2$ (b) distributions in GeV^2/fm^2 for quarks forming an equilateral triangle. Quark separation from the string junction $R = T_g$. Both x_1 and x_3 are measured in T_g. Quark positions are marked with points.

Figure 12: $\langle \mathbf{E}(x_1,x_3)\rangle^2_{3Q}$ (a) and $(\mathbf{E}^{\rm tot}_{3Q}(x_1,x_3))^2$ (b) distributions in GeV^2/fm^2. $R = 5T_g$. Both x_1 and x_3 are measured in T_g. Quark positions are marked with points and vertical lines.

Figure 13: The same as in Fig.12 but for $R = 10T_g$.

Figure 14: The same as in Fig.12(a) but for $R = 30T_g$.

Figure 15: The same as in Fig.12(b) but for $R = 30T_g$.

Figure 16: QQQ string profile $\langle E(0,x_2)\rangle^2_{3Q}$ in GeV^2/fm^2. x_3 is measured in T_g. Dotted, dashed, long dashed and solid lines refer to $R = T_g, 5T_g, 10T_g$ and $30T_g$ respectively.

Figure 17: $\langle \mathbf{E}(x_1,x_3)\rangle^2_{Q-QQ}$ distributions in GeV^2/fm^2 for quark-diquark configuration with $R_Q = 30T_g, R_{QQ} = 0.5T_g$ (a) and $R_Q = 30T_g, R_{QQ} = 3T_g$ (b). Both x_1 and x_3 are measured in T_g. Quark positions are marked with points and vertical lines.

Figure 18: $E_0^2(R) \equiv (\langle E_1(0,0)\rangle^D_{Q\bar{Q}})^2(R)$ distribution in GeV^2/fm^2 as a function of quark separation R measured in T_g.

Figure 19: $\langle E_1(x_1,x_3)\rangle^2_{3Ql}$ distribution in GeV^2/fm^2 for $R_1 = R_2 = 15T_g$ (a) and $R_1 = 10T_g, R_2 = 20T_g$ (b). Both x_1 and x_3 are measured in T_g. Quark positions are marked with points and vertical lines.

Figure 20: Three-quark-interaction potential $V^{(3Q)}(R)$ (a) and the total potential with perturbative one-gluon-exchange included $V^{(3Q)}_{\rm tot}(R)$ (b) as functions of distance R from quarks to string junction measured in T_g.

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